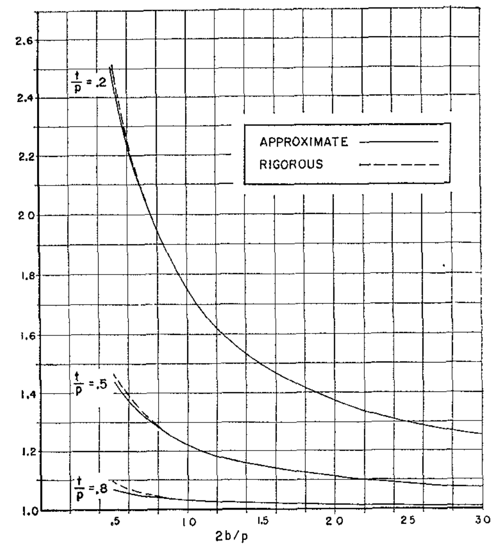

Fig. 13—Theoretical values for the characteristic impedance of multistrip line.

wide strips this quantity is nearly unity. Also included in Fig. 13 is the corresponding result obtained from a rigorous conformal mapping.[7] The rigorous result is too cumbersome in form to be useful in the transverse resonance procedure, but one sees from the comparison in Fig. 13 that the approximate result (27) is extremely accurate over a wide range of parameter values. It is also interesting that in the range in which the two results begin to disagree, the accuracy of the tables required for the rigorous result becomes poor, so that asymptotic expressions must be employed for the functions involved and the rigorous result becomes more difficult to compute from. However, in this range the geometric proportions of the multistrip line are such that other approximations become suitable.

[7] C. A. Hachemeister, "The Impedance and Fields of Some TEM Mode Transmission Lines," Rep. R-623-57, PIB-551, Microwave Res. Inst., Polytechnic Institute of Brooklyn, N. Y.; April 16, 1958.

A Study of a Serrated Ridge Waveguide*

H. S. KIRSCHBAUM† AND R. TSU‡

Summary—**The serrated, or periodically slotted ridge produces a periodic loading which retards the phase velocity of the wave in a waveguide. Such structures may be used to provide a variable index of refraction for microwave lenses and as elements in microwave filters. Two approaches are presented in this paper giving the frequency dependence of the index of refraction. One is based on equivalent circuit representations which are qualitatively valid for the effect of the loading. Circuit parameters which determine the shape of the index of refraction curve are calculated from the experimental data. The other approach providing a purely analytic expression of the index of refraction is derived by a field matching method. Calculations show good agreement with test data.**

Introduction

A SERRATED ridge waveguide is a ridge waveguide with slots cut periodically in the ridge, the slots being transverse to the direction of propagation. The periodicity of the slotting is very small compared to the width of the waveguide, being about 12 per cent of the guide width for those cases studied experimentally.

The purpose of slotting the ridge is to add reactive loading in the guide in order to decrease the phase velocity of the wave. The degree of reduction in phase-velocity depends upon the degree of loading. Such a structure can be used as a means of obtaining a large range of refractive index for use in microwave lenses.[1,2] It may also be used in microwave filter circuitry.

This paper presents a description of two approaches to the evaluation of the index of refraction from the significant dimensions of the guide and its ridge. The first of these is a heuristic approach which seeks to explain the behavior of the guide on the basis of physical reasoning. This leads to two possible transmission line equivalences for the serrated ridge guide, from which the index of refraction can be calculated. This is essentially a semiempirical approach. It enables one to extend the range of knowledge about this structure through the performance of a few judiciously chosen experiments. The second approach involves an attempt to solve the wave equation for the propagation constant in the axial direction of the guide, through the expedient of field

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[1] R. L. Smedes, "High efficiency microwave lens," *Sperry Eng. Rev.*, vol. 9, pp. 1–10; May–June, 1956.

[2] E. K. Proctor, "Methods of reducing chromatic aberration in metal plate microwave lenses," IRE Trans. on Antennas and Propagation, vol. AP-6, pp. 231–239; July, 1958.

matching at certain selected boundaries in the guide. The method is based in part in the selection of an assumed set of hybrid fields to be used in the matching procedure. The method is by no means rigorous and can only be justified by results.

In the case of the first approach, the serrated ridge waveguide is viewed as a ridge waveguide having a short ridge, upon which posts are placed periodically. Since the periodicity is small, it is assumed that the structure is infinitesimally distributed with series *LC* loading across the waveguide. With this assumption an equation for the index of refraction is calculated which contains the static capacitive loading of the posts and the upper cutoff frequency as two parameters. These parameters are found from experimental data. The second transmission line equivalence[3] is viewed as a transmission line corresponding to a ridge waveguide with a full height ridge which is series loaded periodically with short-circuited stubs. The index of refraction calculated from also fits experimental data fairly closely.

In the second approach, two major assumptions are made with regard to the wave numbers in the directions transverse to the guide axis. These are as follows (see Fig. 1):

a) The wave number k_x associated with a pure ridged-guide for the dominant TE mode remains unaltered by the introduction of the slots periodically; and,

b) At frequencies near cutoff, the structure acts as if the slots were not present. In other words it is assumed that the wave number k_x is the same as for the pure ridge case and that k_y is zero at cutoff, taking on values only above cutoff.

In addition a major assumption is made with regard to the nature of the fields in the guide. It is assumed that the transverse field components parallel to the top and bottom faces of the guide are negligible.

Good agreement with experimental data seems to justify these assumptions, and the results so obtained can be put into forms similar to those obtained by the semiempirical approach.

Analysis

Shunt LC Loading

A rectangular waveguide or a ridged waveguide can be represented by an equivalent transmission line insofar as the dominant mode is concerned. The serrated ridge loading structure of Fig. 2 may be looked at in either of two ways. In the first of these, it can be thought of as a ridged guide having a ridge height equal to $(h-d)$. Set upon this ridge are closely spaced posts of height d. A single post partially crossing a waveguide looks very much like a series *LC* structure in the equivalent transmission line representation. Although there is undoubtedly coupling between adjacent posts, this series *LC* circuit was added to the equivalent transmission line of the ridged guide in an attempt to see how closely such a loading element approximates the true situation.

[3] J. R. Pierce, "Travelling Wave Tubes," D. Van Nostrand Co., Inc., New York, N. Y., ch. IV; 1950.

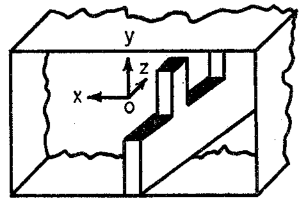

Fig. 1—A section of a serrated ridged waveguide.

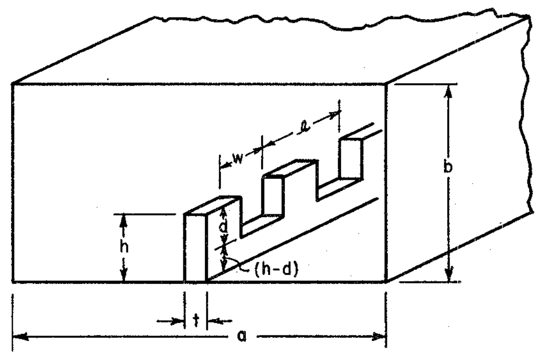

Fig. 2—Serrated ridged waveguide with parameters.

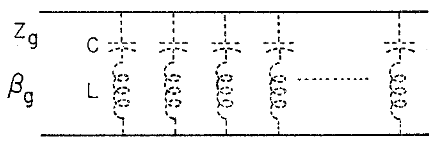

Fig. 3—Equivalent shunt loading due to posts on ridge of height $(h-d)$.

In this representation the loading consists of closely-spaced posts on a ridge of height $(h-d)$. This loading is represented in Fig. 3 as uniformly distributed *LC* sections. In this figure C is the capacitance per unit of axial length between the posts and the upper surface of the guide. The inductance L per unit length is such that its resonance with C accounts for the upper cutoff frequency characteristic of the index of refraction vs frequency curve. The series impedance and shunt admittance per unit length of the equivalent transmission line of the basic ridged guide are

$$Z_{se} = \beta_g Z_g = \frac{2\pi\eta}{\lambda} \tag{1}$$

$$Y_{sh} = \beta_g / Z_g = \frac{2\pi}{\lambda\eta}\left[1 - \left(\frac{\lambda}{\lambda_c}\right)^2\right] \tag{2}$$

where

λ is the free-space wavelength
η is the intrinsic impedance of free space and is μ_0/ϵ_0
λ_c is the cutoff wavelength of the ridged guide of ridge height $(h-d)$.

The shunt loading elements have a shunt admittance per unit length given by

$$Y_{sh}' = \frac{\omega C}{1 - \omega^2 LC} = \frac{2\pi}{\lambda\eta}\cdot\frac{C/\epsilon_0}{1 - (f/f_r)^2} \tag{3}$$

where f_r is the series resonant frequency of the LC loading element. The shunt-loaded guide phase constant is then

$$\beta_g' = \frac{2\pi}{\lambda_g'} = \sqrt{Z_{se}(Y_{sh} + Y_{sh}')}$$

$$= \frac{2\pi}{\lambda}\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2 + \frac{C/\epsilon_0}{1 - (f/f_r)^2}}. \qquad (4)$$

From this the index of refraction is

$$n = \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2 + \frac{C/\epsilon_0}{1 - (f/f_r)^2}}. \qquad (5)$$

In this expression λ_c is the cutoff wavelength of a basic ridged guide of height $(h-d)$ and f_r is the upper cutoff frequency of the loaded ridged guide. Application of this expression to a particular example is shown shortly.

Series Stub Loading

A second way of looking at the loading structure is to view it as a ridge of full height h. Periodically this ridge is series-loaded with short-circuited stubs of length d. The equivalent circuit would be that of the transmission line series loaded with shorted stubs. Again there is probably coupling between stubs, but the simple stub is used to see how closely such a loading element can approximate the true situation.

The stub is characterized by two parameters, the characteristic impedance Z_s of the stub, and the phase constant β_s of wave propagation down and back on the stub. The impedance Z_s is in ohms per unit of length in the axial direction in the guide. Hence if mutual coupling between stubs were negligible, this would be the impedance looking into one stub divided by the distance between stubs.

Eqs. (1) and (2) are applicable to this representation provided the cutoff wavelength corresponding to a ridge height h is substituted for λ_c in (2). To distinguish this cutoff wavelength from that (λ_c) for a ridge height of $(h-d)$, the cutoff wavelength for a ridge height h will be denoted λ_c'. Hence for this case, the stubs have a series impedance per unit of length given by

$$Z_{se}' = Z_s \tan \beta_s d \qquad (6)$$

where β_s is the phase constant for propagation down the slot, and Z_s is the characteristic impedance of the slot per unit of length in the axial direction of the main guide. Thus the index of refraction is

$$n = \frac{\beta_g}{2\pi/\lambda} = \frac{\sqrt{(Z_{se} + Z_{se}')Y_{sh}}}{2\pi/\lambda}$$

$$= \frac{\sqrt{\left(\frac{2\pi\eta}{\lambda} + Z_s \tan \beta_s d\right) Y_{sh}}}{2\pi/\lambda}$$

$$= \sqrt{\left[1 - \left(\frac{\lambda}{\lambda_c'}\right)^2\right]\left[1 + \frac{\lambda Z_s}{2\pi\eta}\tan \beta_s d\right]}. \qquad (7)$$

This expression has the characteristic that for all values of d it gives the same lower cutoff frequency, as contrasted to (5). This difference may be superficial, since at the present time the parameters c, f_r, Z_s, and β_s are obtained by getting the best fit with experimental data.

Example

Both of the above representations were fitted to the case $d/h = 0.700$.[4] The dimensions of the waveguide are listed in Fig. 5. The curves of Figs. 5 and 6 are plotted against frequency normalized to 2.72 kmc. This frequency is the experimentally obtained lower cutoff frequency. This frequency should correspond very closely to the value obtained from λ_c'.

In Fig. 4 there is plotted the experimentally obtained

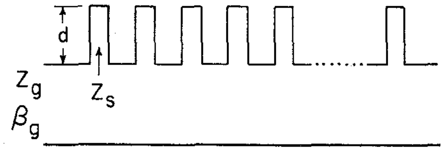

Fig. 4—Equivalent series loading due to stubs in ridge of height h.

function $1/n^2 - 1 + (\lambda/\lambda_c)^2$ against the normalized frequency $(f/f_c')^2$. If (5) is a good representation, this function should be a straight line. As Fig. 5 shows, the representation of a shunt LC loading on a basic ridged guide of ridge height $(h-d)$ is a very good one. The intercepts of this line with the coordinate axes give $\epsilon_0/c = 1.0$ and $(f_r/f_c')^2 = 7.5$. The LC resonant frequency calculated from this is 7.47 kmc, which corresponds to a wavelength of 4.02 cm. The depth of the slot is 0.89 cm, hence the slot is close to being $\lambda/4$ wavelengths long at the upper cutoff frequency.

The "goodness of fit" of the series stub loading cannot be determined as easily as for the shunt LC loading. Here a $\beta_s d'$ was used which made $\beta_s d'$ equal to $\pi/2$ at the upper cutoff frequency. This can be done by using $\beta_s = 2\pi/\lambda$ and $d' = (d +$a correction for end effects), or $d' = d$ and β_s is somewhat greater than $2\pi/\lambda$. In this particular case for $\beta_s = 2\pi/\lambda$, $d' = 1.13\ d$. The value of Z_s cannot be found as easily as the parameters in the case of the first representation.

Fig. 6 shows a plot of n vs (f/f_c') with $Z_s/\eta = 1.2$. This does not necessarily represent the best possible fit. Several values of Z_s should be tried and the best fit chosen perhaps on the basis of minimizing the mean square of the deviation between the calculated curve and the experimental curve.

Both of these representations were tested for a wide variety of ridge and slot dimensions. Over most of the range of dimensional variation these representations gave very good results. However, when the top of the

[4] Experimental data are based on unpublished data (Sperry Reference 7220, 4559; 5440 Series) taken by the Microwave Electronics Div., Sperry Gyroscope Co., Div. of Sperry Rand Corp.

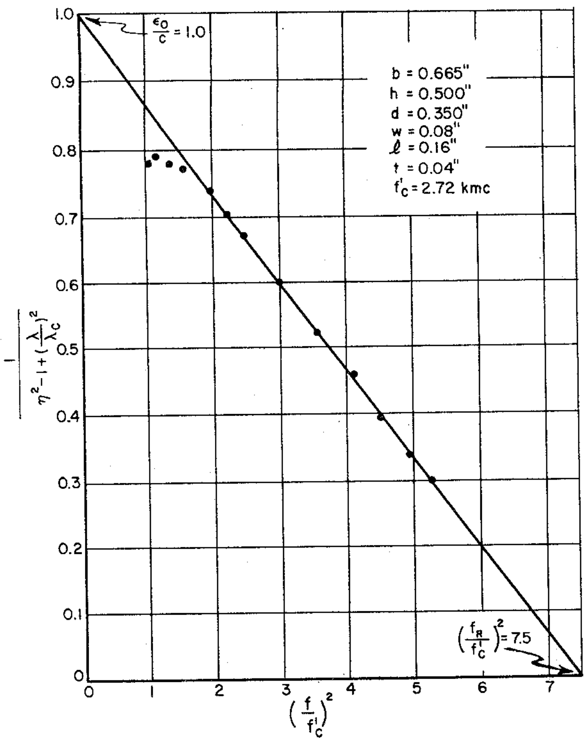

Fig. 5—Variation of

$$\frac{1}{n^2 - 1 + \left(\frac{\lambda}{\lambda_c}\right)^2}$$

with normalized frequency squared.

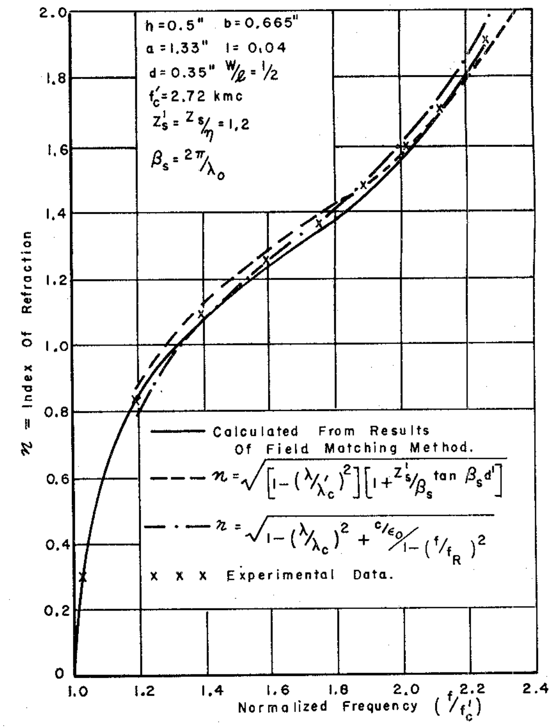

Fig. 6—Comparison of test data with result of calculations.

ridge approach too closely to the top of the guide, the simple equivalent circuits fail. This occurred for h/b greater than about 0.9. Both representations are of doubtful value near the upper cutoff frequency for d/h less than about 0.2, since the upper cutoff frequencies can no longer be accurately determined by extrapolating the shunt LC loading equivalent circuit.

Field Matching Method

The problem of calculating n from the guide dimension, is a boundary value problem with periodic boundary conditions. A rigorous method for this type of problem that leads to an exact and useful solution has not yet been developed.

This section develops a rather simple expression for the propagation constant of the dominant wave based on an assumed approximate set of fields matched over the mutual boundaries of different regions. The method in general is by no means rigorous, and the assumption made can only be justified by results.

Let k_x, k_y and k_z be the wave numbers associated with a solution of the wave equation in the rectangular coordinates with directions respectively as shown in Fig. 1, then

$$k_x^2 + k_y^2 + k_z^2 = k^2 \tag{8}$$

where k is the free space wave number. The two major assumptions are as follows:

a) The wave number k_x associated with a pure ridge guide for the dominant TE mode remains unaltered by the introduction of slots periodically.

b) At cutoff the wave number k_y is zero and takes on values only above cutoff. Good agreement with experimental data seems to have justified the validity of these assumptions.

In order to seek a set of fields that will satisfy the two assumptions, E_x is assumed to be negligible. Due to the periodicity of the boundary, higher order periodic modes are excited[5,6] and the excited waves are periodic in the form of Floquet's solution,

$$F(x, y, z) = \sum_{n=-\infty}^{\infty} A_n F(x, y) e^{j(\omega t - \beta g_n z)} \tag{9}$$

$$\beta g_n = \beta g_0 + \frac{2\pi n}{l} \cdot \tag{10}$$

where βg_0 is the propagation constant of the dominant mode, l is the period of serration, and n is any integer.

Let the entire waveguide in one period be divided into four regions as shown in Fig. 7. Instead of solving the wave equation

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = (\beta g_n{}^2 - k^2) E_z$$

[5] L. Brillouin, "Waveguides for slow waves," *J. Appl. Phys.*, vol. 19, pp. 1023–1041; November, 1948.

[6] L. Brillouin, "Wave Propagation in Periodic Structures," Dover Publications, Inc., New York, N. Y.; 1953.

for an E_z that satisfies all the boundary conditions, an approximate set of fields are chosen which satisfy the wall boundaries and which leave the mutual boundaries open for matching. In the following equations, the left hand subscripts denote the region under consideration.

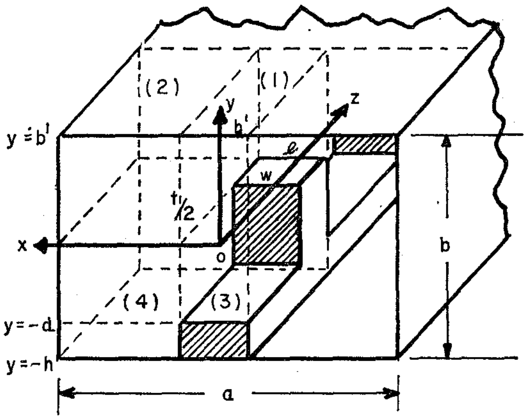

Fig. 7—Showing regions into which guide is divided.

Region (1): $\begin{cases} -t/2<x<t/2 \\ 0<y<b \end{cases}$

Region (2): Outside of (1) and (3), above $y=0$

Region (3): $\begin{cases} -t/2<x<t/2 \\ -d<y<0 \\ 0<z<W \end{cases}$

Region (4): Outside of (1) and (3), below $y=0$.

In region (1), consistent with the assumptions, the fields are

$$_1E_z = \sum_{n=-\infty}^{\infty} A_n \sinh k_{y_n}(b'-y) \cos k_x x e^{-i\beta_{g_n} z}$$

$$_1H_y = \sum_{n=-\infty}^{\infty} \frac{-k_x}{j\omega\mu} A_n \sinh k_{y_n}(b'-y) \sin k_x x e^{-i\beta_{g_n} z}$$

$$_1H_z = \sum_{n=-\infty}^{\infty} \frac{-k_x}{\omega\mu} \frac{\beta g_n}{k_{y_n}} A_n \cosh k_{y_n}(b'-y) \sin k_x x e^{-i\beta_{g_n} z} \quad (11)$$

$$_1H_x = \sum_{n=-\infty}^{\infty} \frac{-K^2}{j\omega\mu} \frac{A_n}{k_{y_n}} \cosh k_{y_n}(b'-y) \cos k_x x e^{-i\beta_{g_n} z}$$

$$_1E_y = \sum_{n=-\infty}^{\infty} \frac{-j\beta g_n}{k_{y_n}} A_n \cosh k_{y_n}(b'-y) \cos k_x x e^{-i\beta_{g_n} z}$$

$$\beta g_n^2 = k^2 - k_x^2 + k_{y_n}^2$$

$$K^2 = k^2\left[1 - \left(\frac{k_x}{k}\right)^2\right]. \quad (12)$$

In (11), the hyperbolic function for the y variation is due to the choice of sign of $k_{y_n}^2$ in (12). The fields in region (2) and (4) can be derived similar to the fields in region (1). Due to the assumption that $k_{y_0}=0$ at cutoff, it can be shown[7] that these fields will lead to a cutoff condition in accordance with that of a pure ridge guide of ridge height h derived by Cohn[8] or Hopfer.[9] In other words the wave number k_x is equal to the cutoff wave number $k_c' = 2\pi/\lambda_c'$.

If a standing wave due to reflection at $Y=-d$, together with higher order attenuated modes, is assumed for the fields in region (3), then

$$_3H_z = C_0 \cos k_x x \sin K(y+d) + \sum_{m=1}^{\infty} C_m \cos\frac{m\pi z}{W} \cos k_x x e^{\gamma_m y}$$

$$_3H_x = -\frac{C_0 K}{j\omega\mu} \cos k_x x \cos K(y+d) + \sum_{m=1}^{\infty} \frac{K}{j\omega\mu} \frac{C_m}{\gamma_m} \frac{m\pi z}{W} \cos k_x x e^{\gamma_m y}$$

$$_3H_y = -\frac{k_x}{j\omega\mu} \sin k_x x \sin K(y+d) + \sum_{m=1}^{\infty} -\frac{k_x}{j\omega\mu} C_m \cos\frac{m\pi z}{W} \sin k_x x e^{\gamma_m y}$$

$$_3H_z = \sum_{m=1}^{\infty} \frac{k_x}{j\omega\mu} \frac{m\pi/W}{\gamma_m} C_m \sin\frac{m\pi z}{W} \sin k_x x e^{\gamma_m y}$$

$$_3E_y = \sum_{m=1}^{\infty} \frac{m\pi/W}{\gamma_m} C_m \sin\frac{m\pi z}{W} \cos k_x x e^{\gamma_m y} \quad (13)$$

where

$$\gamma_m^2 = k_x^2 + \left(\frac{m\pi}{W}\right)^2 - k^2.$$

For $m>0$, if the period is much smaller than the wavelength, γ_m becomes real and is approximately $m\pi/W$. This means the higher order modes are attenuated rapidly, as they travel down into the slot and is the reason reflection at $y=-d$ is not required for $m>0$ in (13).

Going back to the guide constant βg_n, for $n \neq 0$, $\beta g_n \approx 2\pi n/l$. These rapidly phase changing modes are essential in order to satisfy the periodic boundary condition.

To match the fields in region (1) with the slot wave, a variational technique is used. The fields $_1E_z$ and $_3E_z$ must be equal to a field E defined over the common boundary from 0 to W, and $_1H_x$ must be equal to $_3H_x$ over the slot opening. Together with (11) through (13), there results

[7] R. Tsu, "Analysis of a Serrated Ridged Waveguide," Antenna Lab., The Ohio State Univ. Res. Found., Rep. 744-4; December 31, 1957.

[8] S. B. Cohn, "Properties of ridge waveguide," PROC. IRE, vol. 35, pp. 783–788; August, 1947.

[9] S. Hopfer, "The design of ridged waveguides," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-3, pp. 20–29; October, 1955.

$$\frac{\coth k_{y_0}b'}{k_{y_0}} = \frac{\frac{l}{W}\left\{\frac{\cot Kd}{K}\int_0^W E dz \int_0^W E^* dz - \sum_{m=1}^{\infty}\frac{2}{\gamma_m}\int_0^W E\cos\frac{m\pi z}{W}dz\int_0^W E^*\cos\frac{m\pi z}{W}dz\right\}}{\int_0^W E^{i\beta g_0 z}dz\int_0^W E^* e^{-i\beta g_0 z}dz}$$
$$- \frac{\sum_{n=-\infty}^{\infty\,\prime}\frac{\coth k_{y_n}b'}{k_{y_n}}\int_0^W E e^{i\beta g_n z}dz\int_0^W E^* e^{-i\beta g_n z}dz}{\int_0^W E e^{i\beta g_0 z}dz\int_0^W E^* e^{-i\beta g_0 z}dz}. \quad (14)$$

Eq. (14) is in the variational form such that $\coth k_{y_0}b'/k_{y_0}$ is stationary with respect to small variation of E from its corrected value.[10] $\sum_{n=-\infty}^{\infty\prime}$ denotes summation from $-\infty$ to ∞ without the term $n=0$. Similarly another variational form in terms of the H field can be formulated, however, results will be obtained from (14).

A very simple expression results if the trial field is taken to be unity, or $E=1$. In this case (14) gives

$$\frac{l}{W}\frac{\cot Kd}{K} - \sum_{n=1}^{\infty}\frac{l^3\coth\frac{2\pi nb'}{l}}{2\pi^3n^3W^2}\left(1-\cos\frac{2\pi nW}{l}\right) - \frac{\coth k_{y_0}b'}{k_{y_0}} = 0. \quad (15)$$

$\beta g_n \approx 2\pi n/l$ for $n\neq 0$ is used to obtain (15).

To identify the meaning of each term in (15), it is interesting to note that the first term is the normalized admittance of the slot, the last term is the normalized admittance due to the fields above the teeth, and the middle term can be regarded as a discontinuity admittance y_1 due to a step in the y-direction as shown in Fig. 8(a). Eq. (15) does not give satisfactory results since the trial field of unity drops out the term involving summation over m. According to Fig. 8(b), we can identify this missing term as being the step discontinuity admittance y_2 due to a step in z-direction. Fig. 8(c) represents a step discontinuity admittance y_3 due to a step in x-direction if the field E_x were not assumed to be zero. Hence if the rest of the discontinuity admittances be added to (15), there results

$$\frac{l}{W}\frac{\cot Kd}{K} - \sum_{n=1}^{\infty} l\left(\frac{l}{W}\right)^2 \frac{\coth\frac{2\pi nb'}{l}}{2\pi^3n^3} \cdot\left(1-\cos\frac{2\pi nW}{l}\right) - y_2 - y_3 = \frac{\coth k_{y_0}b'}{k_{y_0}}. \quad (16)$$

To find y_2, the discontinuity capacitance per unit length for a symmetrical step in z is $C_2=(\epsilon_0/\pi)\ln\csc(\pi W/2l)$. Thus the admittance y_2 normalized with respect to $-j\omega\mu l/K^2W$ is

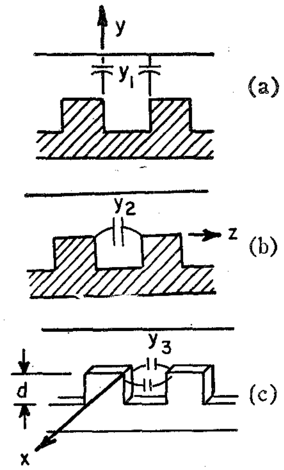

Fig. 8—Discontinuity capacitances y_1, y_2 and y_3.

$$(j\omega C_2 l)\left(\frac{-j\omega\mu}{K^2}\right)\frac{l}{W}.$$

The factor l/W appears for all the quantities referring to the slot [see (16)]. Similarly, for the case shown in Fig. 8(c), the discontinuity capacitance per unit length $C_3=C_2$, and the admittance y_3 normalized with respect to

$$\left(\frac{l}{W}\right)\frac{-j\omega\mu}{K^2},$$

is given by

$$\left(\frac{l}{W}\right)\left(\frac{-j\omega\mu}{K^2}\right)j\omega(2C_3l)kd.$$

The factor two in front of C_3 is due to two step discontinuities at $x=\pm t/2$ in shunt, and the factor kd takes into account the slot depth because the total step discontinuity consists of all such elements in shunt from $y=0$ to $y=-d$. These normalizations are required because of the form of the admittances appearing in (16). Therefore

$$y_2 + y_3 = \frac{l}{W}\frac{(C_2/\epsilon_0)l}{1-\left(\frac{k_x}{k}\right)^2} + \frac{l}{W}\frac{(2C_3/\epsilon_0)lkd}{1-\left(\frac{k_x}{k}\right)^2} \quad (17)$$

where $C_2=C_3=(\epsilon_0/\pi)\ln\csc(\pi W/2l)$ farads/meters. Rearranging (16), there results

[10] L. Lewin, "Advanced Theory of Waveguides," Iliffe & Sons Ltd., London, Eng.; 1951.

$$k_{y_0} \tanh k_{y_0} b' = \frac{W}{l} K \tan Kd \frac{1}{1 - \left[\sum_{n=1}^{\infty} l \left(\frac{l}{W}\right) \frac{\coth \frac{2\pi n b'}{l}}{2\pi^3 n^3} \left(1 - \cos \frac{2\pi n W}{l}\right) + \frac{\frac{C_2}{\epsilon_0} l}{1 - \left(\frac{k_x}{k}\right)^2} + \frac{2 \frac{C_2}{\epsilon_0} lkd}{1 - \left(\frac{k_x}{k}\right)^2} \right] K \tan Kd} \tag{18}$$

where

$$K = k\sqrt{1 - \left(\frac{k_x}{k}\right)^2}$$

$$\beta g_0 = k\sqrt{1 - \left(\frac{k_x}{k}\right)^2 + \left(\frac{k_{y_0}}{k}\right)^2}$$

$$k_x = k_c = 2\pi f_c' \sqrt{\mu_0 \epsilon_0}.$$

From (18), the propagation constant of the dominant mode can be calculated. Note that the term in the braces can easily be calculated for each given k, and a simple graph can be used to determine k_{y_0}. It is interesting to note that the index of refraction

$$n \equiv \frac{\beta g_0}{k} = \sqrt{1 - \left(\frac{k_x}{k}\right)^2 + \left(\frac{k_{y_0}}{k}\right)^2} \tag{19}$$

is of the same form as the shunt LC loading formula, and with a little rearranging, (19) becomes

$$n = \sqrt{1 - \left(\frac{k_x}{k}\right)^2 + \left(\frac{k_{y_0}}{k}\right)^2}$$

$$= \sqrt{\left\{-1\left(\frac{k_x}{k}\right)^2\right\} \left\{1 + \frac{k_{y_0}^2}{k^2\left[1 - \left(\frac{k_x}{k}\right)^2\right]}\right\}}$$

$$= \sqrt{\left[1 - \left(\frac{k_x}{k}\right)^2\right]\left[1 - \left(\frac{k_{y_0}}{K}\right)^2\right]}$$

which is identical in form to (7) derived from series stub loading consideration. The index of refraction calculated from (18) for the example given previously is also plotted in Fig. 6.

Conclusions

The equivalent circuit representations give a good physical picture for the effect of the loading although they are only qualitatively correct. Effort to correlate the loading parameters with the dimensions of the structure was made; however,[11] no simple relation seems to exist. On the other hand, the field matching results give the desired accuracy. It is worth noting that a better trial field for the variational expression, for example, a field that symmetrically goes to infinity at the extremities of each slot, may be used. Further improvements, if desired, have to be made with a better set of hybrid fields without neglecting the field E_x [see discussions prior to (16)].

[11] H. Kirschbaum and R. Tsu, "A Study of a Serrated Ridged Waveguide," Antenna Lab., The Ohio State Univ. Res. Found., Rep. 744-2; July 31, 1957.